A Study About Anchor Mooring Calculations – Part 1 (Basics)

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This preprint discusses the fundamental principles of anchor mooring, providing the basic formulations and its solution. In this Part 1, we limit the study to a homogeneous (constant linear weight), flexible (non-rigid), and inelastic (non-stretching) mooring cable (or rope) for the operational load range[3], and assume that the water bed is plane and horizontal. There will be no information about anchors, namely: their principles, engineering, uses, and capacity. We assume that, within the operational load range of the system, the anchor will not drag and the tangent point of contact of the mooring cable never reaches the anchor. The final goal is to obtain the total tension the chain exerts onto the vessel's mooring winch (winch, for short). This anchoring system is typical on large ships, offshore drilling and production vessels, and on some marine renewable energy systems. Offshore rigs in shallow to mid deep waters may employ four up to 16 anchors in various lay-outs to keep the vessel's station during operations[4]. That said, let's see the equations governing the anchor mooring mechanics. The derivation of these equations is in Appendix 1. The source code used in this preprint is available as OSS.[5]

Figure (1) shows the geometry of an anchoring system in an operational configuration.[6] At this configuration, the geometric and physical parameters under control (up to a point) are:

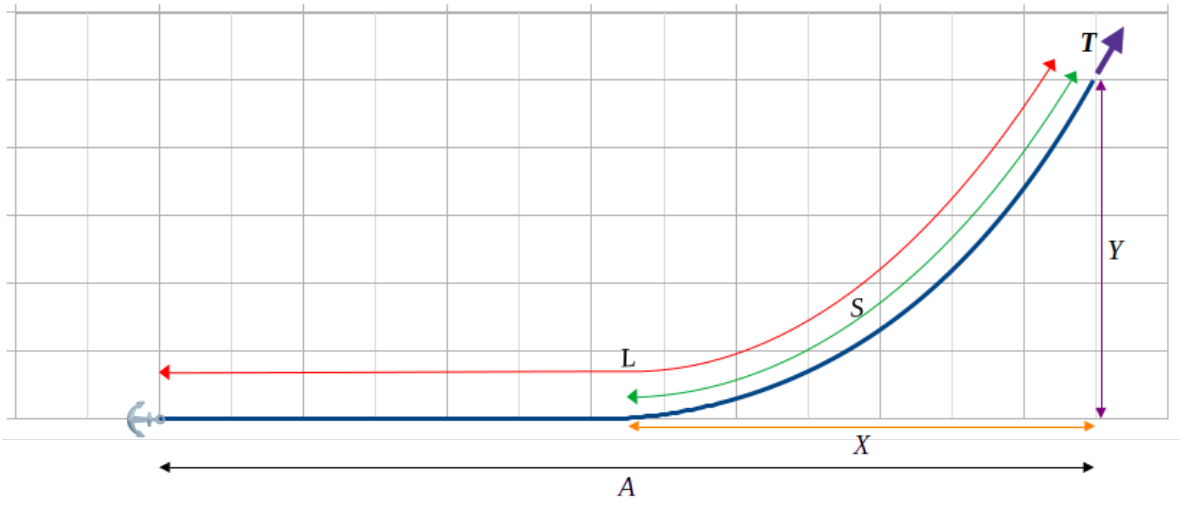

Figure 1

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2 ↄ is the Copyleft symbol. If you don't know it, look here: https://en.wikipedia.org/wiki/Copyleft and here https://copyleft.org

3 A chain or heavy steel rope.

4 In deep waters, dynamic positioning systems, tether mooring, or elastomeric mooring replaces the common anchor mooring.

5 Source code here: https://github.com/jrgsampaio/OilAndGas/tree/Mooring-Systems. Please note CITATION.cff.

6 How this configuration is reached, and with which purpose, will not be discussed now.

- Y - depth of the water – assumed constant for now, but sensitive to tides, weather, etc.,
- A - horizontal distance of the anchor to the winch – assumed constant for now but can change due to external factors like wind, currents, etc.,
- L - total length of the mooring cable from winch to the anchor – assumed constant for now, although the mooring cable can be reeled in and out by the winch,
- w - the linear weight of the cable – assumed constant along the mooring cable, from the winch to the anchor.

Beside those parameters, we also have

- S - the length of the suspended cable – the portion of the mooring cable that is not lying on the bottom,
- X - the horizontal distance from the vessel to the tangent point, that is, the point that the hanging cable firstly touches the bottom,
- Y' - the slope of the cable at the vessel's mooring,
- T - the tension load at the vessel's mooring – this is the force acting on the mooring cable by the vessel (and vice versa),
- H and V - the horizontal and vertical components of T,
- c - an initially unknown parameter, not constant.

Based on these parameters we have the following equations:

$$Y = c\left(\cosh\frac{X}{c} - 1\right) \tag{1}$$

$$S = c\sinh\frac{X}{c} \tag{2}$$

$$Y' = \sinh\frac{X}{c} = \frac{H}{V} \tag{3}$$

$$T = \sqrt{H^2 + V^2} \tag{4}$$

$$c = \frac{H}{w} \tag{5}$$

Also, from Figure (1), we can verify that the following relation exists:

$$L - A = S - X \tag{6}$$

These equations come from the theory of catenary, presented in Appendix 1. Next we will discuss a way of solving these equations to conceive a single point anchor mooring system, and simulate and analyze some cases. The tension T has two antagonistic effects. The horizontal component H has a a beneficial effect because it reacts to external forces due to currents and winds and keeps the vessel under control and in position. On the other hand, the vertical component V has a detrimental effect because it loads the vessel (dead load), pulling it downwards and reducing its available capacity to carry live loads. Because these two force components are intrinsically linked, our interest is to define the controlled parameters, within their range, for the best balance between H and T.

A first concern about the equations above, specifically Equations (1) and (2), is that we can not solve them algebraically to obtain the parameter c. We need c to calculate the other parameters of interest. This because Eq.(1) and Eq.(2) are transcendental[7], i.e., not algebraic equations. Besides,

7 We can solve some transcendental functions using a particular transformation, like $4^x = 3^{x^2-1}\cdot 2^{5x}$. Few are immediate like $2^x = x^2$. Some others, like $x = e^x$, $x = \tan x$, and particularly $x = \sinh x$ requires numerical solutions.

they include the unknown X, which requires to calculate S from Eq.(4). But we need to obtain c and, for this endeavor, we must use numerical, iterative solution.

In fact, there are several ways of doing that. The principle, however, is the same: we pick an unknown transcendental parameter (one that appears inside and outside of a non-algebraic function) and guess an initial value for it (hopefully a lucky one); then use one or more of the equations to recalculate a new value for the parameter and compare it to the old one. Eventually the sequence of new values converges; sometimes it doesn't.

However, each of the equations above has two or more unknown parameters. For example, we can pick Eq.(1), which contains two unknown parameters, and guess initial values for c and X. Then, keeping c constant, iteratively find X that solves Eq.(1) and use this value of X in Eq.(2) to recalculate c, and iterate the process until both c and X converge simultaneously. This process eventually works, but it can take a large number of iterations and, sometimes, it may not converge. I have tried several approaches with various levels of success, and non-success. The approach I found with the highest value of success, and quite stable and convergent, is the following. Consider Eq.(6) and replace S by Eq.(2)

$$L-A=S-X$$

$$L-A=\left(c\sinh\frac{X}{c}\right)-X$$

$$L-A=c\left(\sinh\frac{X}{c}-\frac{X}{c}\right)$$

Now, properly divide this equation by Eq.(1) to obtain

$$\frac{L-A}{Y}=\frac{\sinh\frac{X}{c}-\frac{X}{c}}{\cosh\frac{X}{c}-1}$$

Note that doing this, the parameter c is eliminated from outside the transcendental functions. Note also that $\frac{(L-A)}{Y}=k$ is known, since L, A, and Y are controlled parameters. Making $\xi=\frac{X}{c}$ we reach at

$$k=\frac{\sinh\xi-\xi}{\cosh\xi-1} \tag{7}$$

Note that this equation encapsulates all the problem's pertinent parameters. All we need now is to seek one or more particular values of ξ that solves this equation. Possibly, an efficient way of solving it is using the Newton-Raphson (N-R) method (or any of its variation). For that, let's define the following function

$$f(\xi)=\frac{\sinh\xi-\xi}{\cosh\xi-1}-k \tag{8}$$

For proper values of ξ (called roots) $f(\xi)=0$, and this is what we are after in Eq.(7). This function is valid, in principle, for the whole real axis ($\mathbb{R}$). However, for computer calculations, $\xi=0$ will result in an error, because $\cosh 0=1$. However, the numerator will also be zero, and using the L'Hôpital's rule one can easily find that $\lim_{\xi\to 0} f(\xi)=-k$. Plotting this function for $k=0.4$ we get:

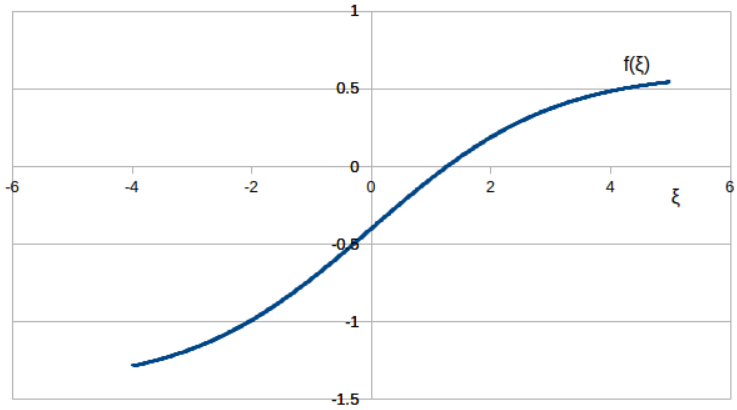


Figure 2

We can see that, for this particular value of k, a root occurs at about $\xi \approx 1.26$. Note that since k is always positive, this root will also be positive. (The effect of k is to drag the graph downwards.)

To apply the Newton-Raphson method, we also need the derivative of $f(\xi)$, formally obtained and given by

$$f'(\xi) = 1 - \frac{\sinh\xi(\sinh\xi - \xi)}{(\cosh\xi - 1)^2} \tag{9}$$

This derivative function is also valid in the whole real axis ($\mathbb{R}$). However in computer calculations, $\xi = 0$ it will result in error, because $\cosh(0) = 1$ (squared). Being the numerator also zero, and using the L'Hôpital's rule we can (not so easily) find that $\lim_{\xi \to 0} f'(\xi) = 1/3$. Plotting the derivative for $k = 0.4$ we obtain (on top of the previous):

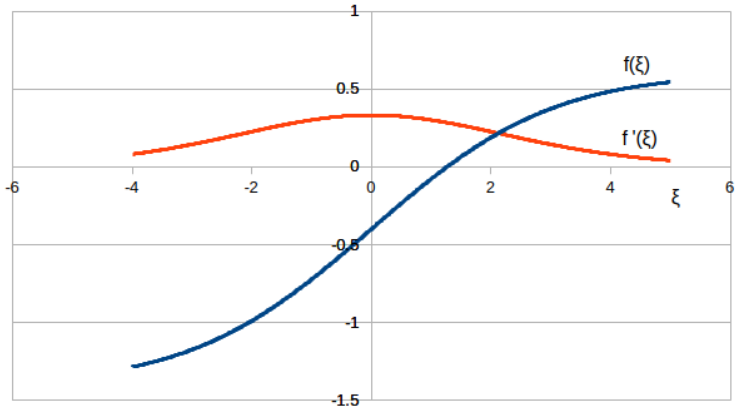


Figure *3*

We can see from the graphs of $f(\xi)$ and $f'(\xi)$ that a Newton-Raphson method can throw the next value of ξ on the other side of the root and eventually far from the root, particularly if the initial guess is larger than the root. A safer initial guess is one smaller than the root, and a very reasonable initial guess for the root is $\xi_0 = 3k$.[8] This indicates that the scheme provided by Eq.(9) can be robust. In fact, a simulation where $k = 0.99998^9$ required 14 iterations to find the result. Generally, within the typical operational range it takes 3 to 5 iterations to converge to the correct root.

8 If this seems obscure, see Appendix 2, which discuss the Newton-Raphson method of root finding.

Before we calculate the remaining parameters, some considerations are needed. The first consideration is that, in Eq.(8), $f(\pm\infty)=\pm 1-k$. This means that $f(\xi)$ is bounded. A second consideration is that, from Eq.(9), $f'(\xi)$ is everywhere positive and asymptotically to zero in both branches of $\mathbb{R}$. This means that $f(\xi)$ has one and only one root. That is, any root finding algorithm will either find the single root, or diverge to $\pm\infty$.

After finding the root ξ, we can calculate the other parameters. From Equations (1) and (3) we obtain c and Y':

$$c=\frac{Y}{(\cosh\xi-1)} \tag{10}$$

$$Y'=\sinh\xi \tag{11}$$

With c we obtain H, X, and S:

$$H=c\ w \tag{12}$$

$$X=c\,\xi \tag{13}$$

$$S=c\sinh\xi \tag{14}$$

And with Y' and H we obtain V and T:

$$V=Y'H \tag{15}$$

$$T=\sqrt{H^2+T^2} \tag{4}$$

Before we jump into an example and analysis, we still need to discuss a last detail. For given depth Y and anchor distance A, there is a maximum and minimum length for the mooring cable L. The maximum length is given by

$$L_{max}=Y+A$$

At this condition, the cable hangs vertically from the winch to the bottom and from there it extends horizontally all the way to the anchor. The minimum length occurs when the tangent point, given by the distance X, is at the anchor location, i.e., $X=A$. If we shorten the cable less than this length, the anchor can break out (unhook from the bottom).

The approach to calculate L_{min} is similar but simpler to what we've done before. Considering Eq.(1) and the fact that $X=A$ we get

$$Y=c\left(\cosh\frac{A}{c}-1\right)$$

$$\frac{1}{c}=\frac{1}{Y}\left(\cosh\frac{A}{c}-1\right)$$

Multiplying both sides by A and making $\frac{A}{c}=\zeta$ gives

$$\frac{A}{c}=\frac{A}{Y}\left(\cosh\frac{A}{c}-1\right)$$

$$\zeta=\frac{A}{Y}(\cosh\zeta-1)$$

9 In fact, $k=1$ represents the limit case in which no external force acts on the vessel and $L-A\Rightarrow Y$, that is, the vessel is dragged towards the anchor until it lays right above the tangent point. (See the discussion about L_{max} next.)

To find ζ we use N-R and the following function

$$g(\zeta)=\frac{A}{Y}(\cosh\zeta-1)-\zeta$$

noting that

$$g'(\zeta)=\frac{A}{Y}\sinh\zeta-1$$

After ζ is found, we calculate the minimum length using the definition of ζ and Eq.(2):

$$L_{min}=\frac{A}{\zeta}\sinh\zeta \tag{16}$$

Now, let's see an illustrative example, and do some analyses. We consider a vessel floating on waters with depth of 500 m[10]. An anchor is launched 1400 m away from the vessel and the cable is reeled in until the cable is 1700 m long. We want to know the horizontal and vertical forces acting on the vessel due to the weight of the cable. Let's assume that the linear weight of the cable is 262 kgf/m[11].

For these mooring parameters, $k=\frac{L-A}{Y}=\frac{1700-1400}{500}=0.6$. Starting with $\xi_0=3k=1.8$, and using Eqs. (8) and (9) we have:

$$f(1.8)=\frac{\sinh 1.8-1.8}{\cosh 1.8-1}-0.6=-0.058\ 036\ 144\ 45$$

$$f'(1.8)=1-\frac{\sinh 1.8(\sinh 1.8-1.8)}{(\cosh 1.8-1)^2}=0.243\ 382\ 008\ 95$$

Using the N-R formula (see Appendix 2) the next trial for ξ is

$$\xi_1=1.80-\frac{-0.058\ 036\ 144\ 45}{0.243\ 382\ 008\ 95}=2.038\ 457\ 003\ 07$$

And the next trial for ξ is

$$f(2.038\ldots)=\frac{\sinh 2.038\ldots-2.038\ldots}{\cosh 2.038\ldots-1}-0.6=-0.002\ 372\ 274\ 9$$

$$f'(2.038\ldots)=1-\frac{2.038\ldots(\sinh 2.038\ldots-2.038\ldots)}{(\cosh 2.038\ldots-1)^2}=0.223\ 408\ 376\ 76$$

$$\xi_2=2.038\ldots-\frac{-0.002\ 372\ 2749}{0.22340837676}=2.049\ 075\ 561\ 49$$

Continuing for 2 more iterations we get:

$$f(2.049\ldots)=\frac{\sinh 2.049\ldots-2.049\ldots}{\cosh 2.049\ldots-1}-0.6=-0.000\ 004\ 767\ 35$$

$$f'(2.049\ldots)=1-\frac{2.049\ldots(\sinh 2.049\ldots-2.049\ldots)}{(\cosh 2.049\ldots-1)^2}=0.222\ 510\ 380\ 96$$

$$\xi_3=2.049\ 096\ 986\ 84$$

10 This is very typical in Oil&Gas offshore operations.

11 I am using a standard offshore Grade 3 cable (studded) with linear weight of 300.7 kgf/m in seawater. The buoyancy factor for steel in seawater is approximately 0.87. One can find specifications of marine cables here: https://www.jeyco.com.au/catalogues.html

$$f(2.049\ldots)=\frac{\sinh 2.049\ldots-2.049\ldots}{\cosh 2.049\ldots-1}-0.6=-0.000\ 004\ 767\ 36$$

$$f'(2.049\ldots)=1-\frac{2.049\ldots(\sinh 2.049\ldots-2.049\ldots)}{(\cosh 2.049\ldots-1)^2}=0.222\ 508\ 568\ 66$$

$$\xi_4=2.049\ 096\ 986\ 93$$

The absolute fractional error between ξ_4 and ξ_3 is

$$\left|\frac{\xi_4-\xi_3}{\xi_3}\right|=\left|\frac{\xi_4}{\xi_3}-1\right|=4.392\times10^{-11}\ll10^{-6}$$

In fact, if we plug ξ_4 into Eq.(2) we get

$$f(\xi_4)=\frac{\sinh 2.049\ 096\ 986\ 93-2.049\ 096\ 986\ 93}{\cosh 2.049\ 096\ 986\ 93-1}-0.6=8.43\times10^{-13}$$

which confirms that the process did find the root. With ξ we are able to find the dependent parameters of the example:

$$c=\frac{500}{(\cosh 2.049\ 096\ 986\ 93-1)}=169.786\ 754\ \text{m}$$

$$Y'=\sinh\xi=\sinh 2.049\ 096\ 986\ 93=3.816\ 019\ 3$$

$$H=c\ w=169.786\ 754\times262=44{,}484\ \text{kgf}$$

$$X=c\ \xi=169.786\ 754\times2.0491=347.9\ \text{m}$$

$$S=c\sinh\xi=169.786\ 754\times\sinh 2.049\ 096\ 986\ 93=647.9\ \text{m}$$

$$V=Y'H=3.816\ 019\ 3\times44{,}484=169{,}752\ \text{kgf}$$

$$T=\sqrt{44{,}484^2+169{,}752^2}=175{,}484\ \text{kgf}$$

For effect of illustration, the geometric configuration of the cable is shown in Figure (4).

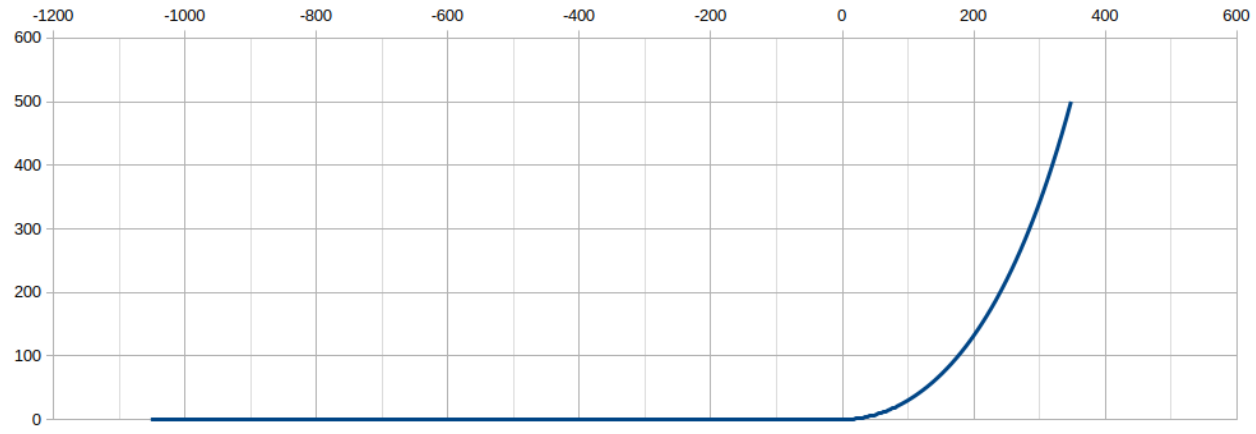


Figure 4

In this graph (with a relatively homogeneous scale), the origin of the axes are in the tangent point of the cable. We can find the vessel's mooring winch some 350 m to the right, and the anchor some 1050 m to the left ($A=1400$ m). We can also verify the slope of the cable at the vessel's mooring

winch ($Y' = 3.816$). This slope results in a vertical force $V = 170$ MT[12] much larger than the horizontal force $H = 44.5$ MT.

Also for illustration, Figure (5) shows the situation when the length of the cable is set to 1889 m (1 meter less then the maximum).[13]

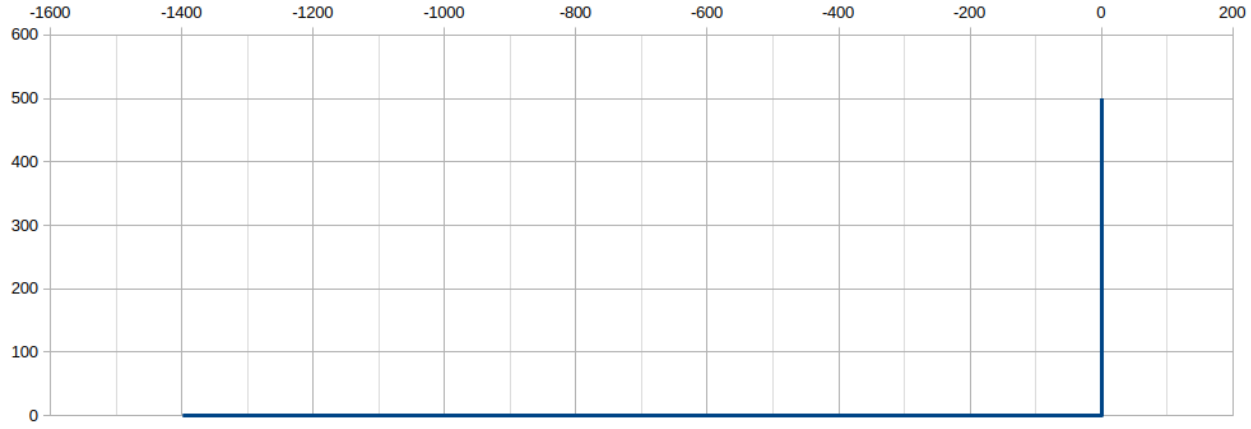


Figure 5

Note that in this case, as expected, the cable hangs (almost) vertically from the winch to the tangent point and, from that point it extends horizontally all the way to the anchor. On the other hand, if the length of the cable is set to 1512.70 m[14], the graph is as in Figure (6).

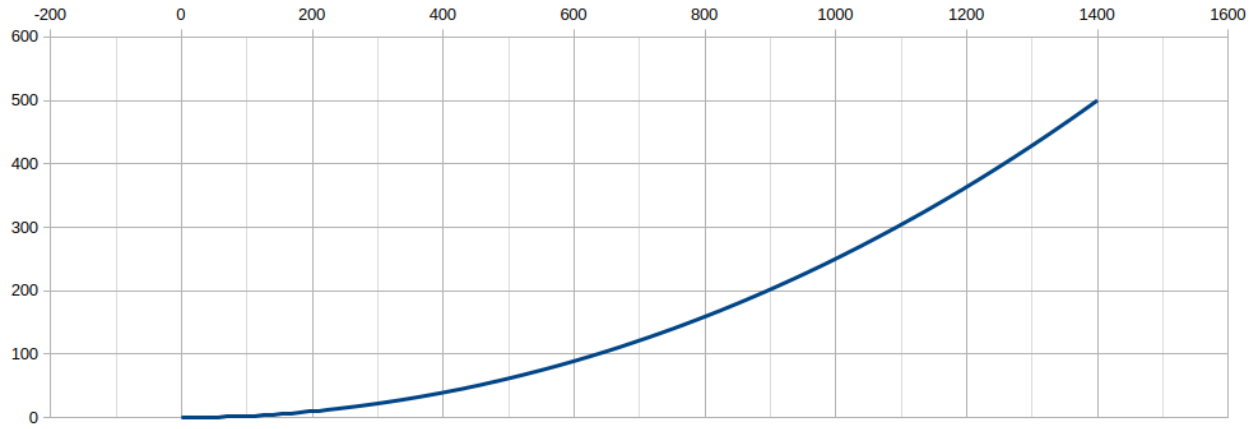


Figure 6

Note that the tangent point occurs right at the anchor (but still tangent to the bottom at this point).This means that there is no vertical component of the cable tension at the anchor

Now we will consider some dynamics of a single point anchor. In a situation like above the result of the horizontal force H acting on the winch is to pull the vessel closer the anchor (if no other

12 "MT" here is an abbreviation for *metric ton,* or *tonne* = 1,000 kgf = 2,201.6 lbf.

13 This results from the actual calculation with $L = 1899$ m. If 1900 m is used, the code can not calculate it.

14 This is the actual minimum length calculated with the procedure described in the text. If a shorter length is entered, the code still produces a number but it will not represent a real physical result. In fact, simple anchors are not designed to withstand any safe vertical force at the point it is hooked on the water bottom.

external force is acting on the vessel) until the vessel is over the tangent point, for the set length of the cable. In the case, this will occur then the vessel is 1200 m from the anchor.[15]
This configuration is shown in Figure (7). On the other hand, if an external effect acts on the vessel (wind, water current, etc., or any combination of those) resulting on a horizontal force of 44.5 MT, the cable will assume the configuration shown in Figure (4).

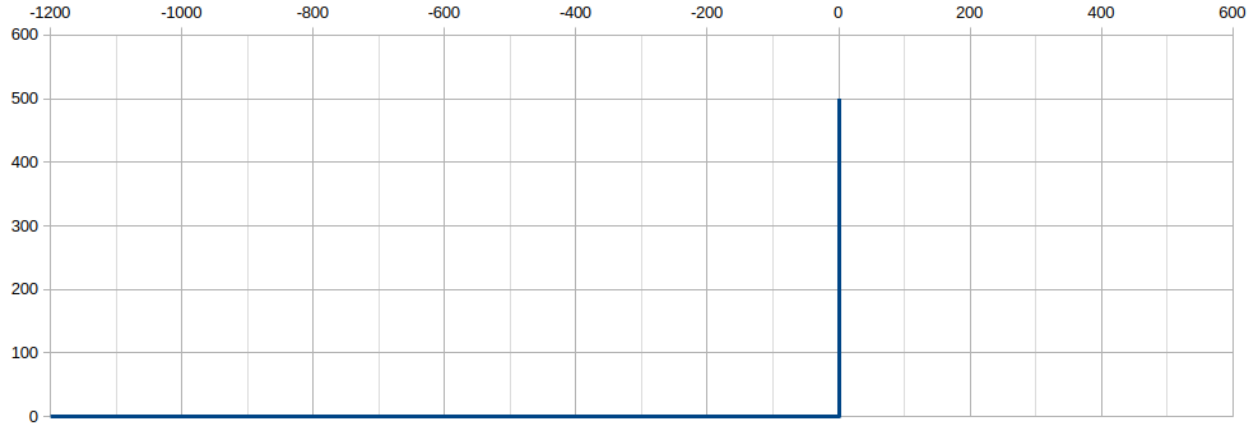


Figure 7

Now, what happens if the resulting force acting on the vessel changes to 59.7 MT? The consequence is that the vessel will move to the right (increasing the distance A to the anchor) until the horizontal force balances the external force. The result of this effect is shown in Figure (8), where the portion in red is the configuration of the suspended cable, compared with that of Figure (2) (in blue). The axes' origins of this new graph is at the new tangent point, which has moved left with respect with the original position.

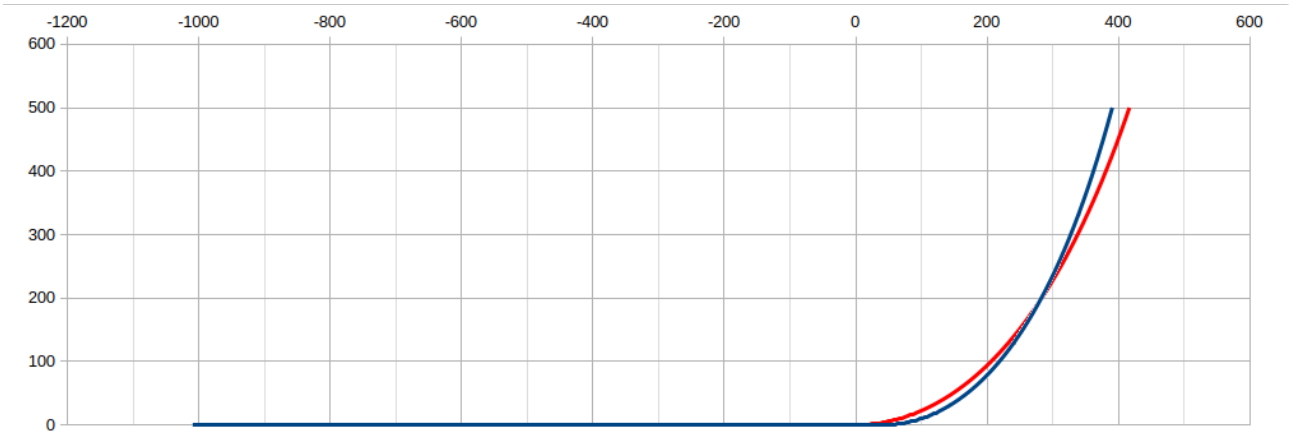


Figure 8

Calculations show that now the vessel is now 1425.5 m from the anchor and the suspended length of the cable is 672.5 m.

On the other hand, if the resulting external force acting on the vessel drops to 33.5 MT, the vessel will be pulled to the left until the horizontal force balances this new external force. This is shown Figure (9) where the portion in green corresponds to this new external force. Note that the new tangent point (which is at the axes' origins in the graph) has moved to the right of the original point,

15 $L - Y = 1700 - 500 = 1200$ m

and the vessel has moved to the left. Calculations show that the vessel is now 1376.1 m from the anchor, and the suspended length of the cable is 614.8 m.

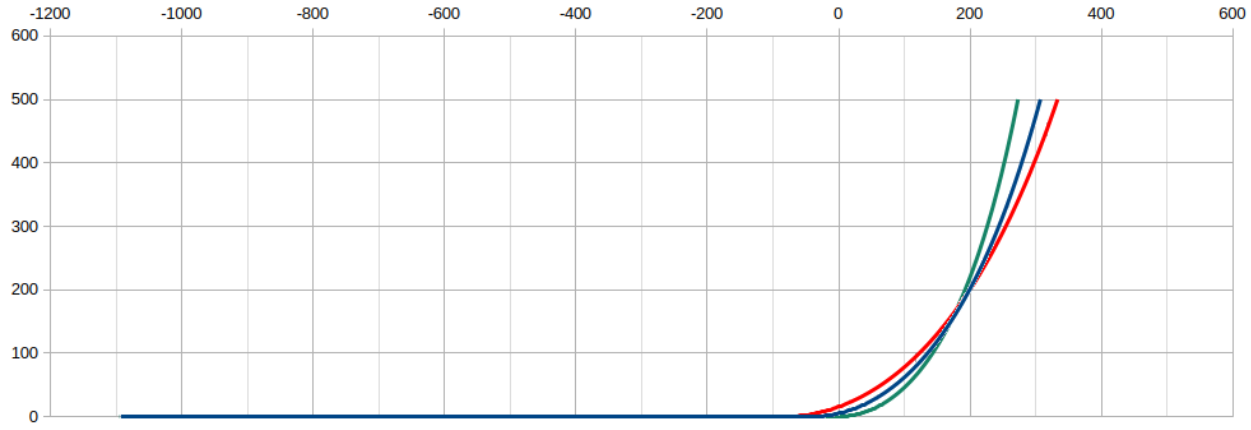


Figure 9

From these analyses we can realize that a single point mooring system, with the configuration above, can allow a considerable drift of the vessel, from a calm environment ($A = 1200$ m) to $A = 1425$ m or even further. If is desired to keep the vessel within a particular location, a single point mooring anchor may not be satisfactory. For example, if a vessel must be kept within a restricted distance to its designed station, a second anchor may be used opposite to the first with respect to the vessel. But even in this case of two opposite anchors can be very inefficient when the resulting external force is perpendicular to the plane containing the anchors, cables and we must have to use more than two anchors. The dual point and multiple points anchor mooring will be discussed in a following preprint.

Appendix 1 – Derivation of the Mooring Chain's Formulas (Catenary)

Figure (10) shows a free-body diagram of an infinitesimal length dS of a suspended line subjected to its own weight. It shows at x the vertical position y, the tension force T and the slope y', and at $x+dx$ the vertical position $y+dy$, the tension $T+dT$ and the slope $y'+dy'$. It also shows the weight $w\,dS$ of the infinitesimal length of line dS. One immediate conclusion from this diagram is that the horizontal components of T and $T+dT$ must be equal, that is, the horizontal component H of the chain's tension is constant along its length.[16] This results immediately from the equilibrium of forces acting on the free body diagram. We conclude, also from the equilibrium, that the differential dV of the tension's vertical component is equal to the weight of the dS:

$$dV = w\,dS$$

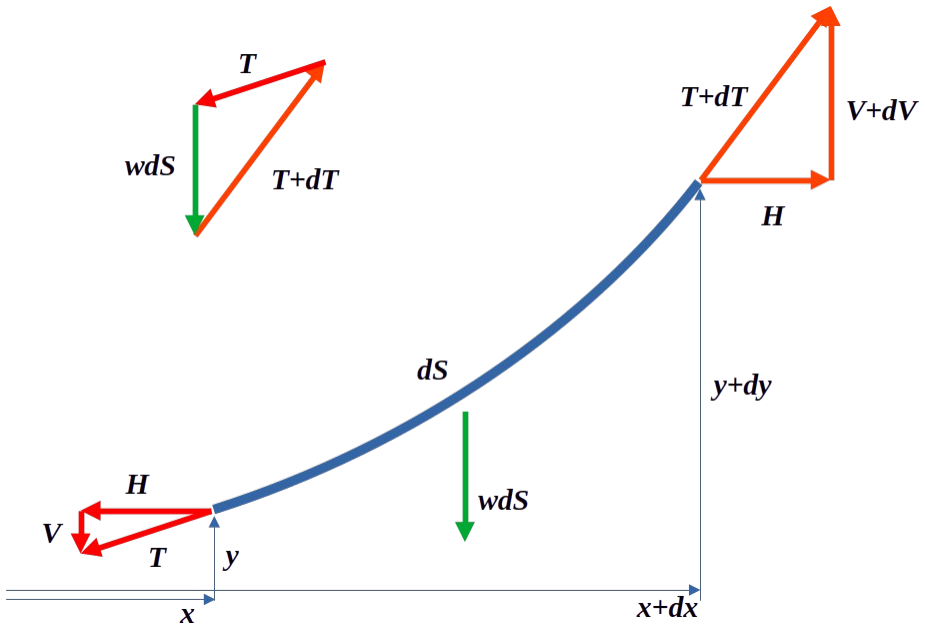


Figure 10

In addition, from this diagram the we have that

$$y'(x) = \frac{-V}{-H} = \frac{V}{H}$$

Since H is constant, $dy' = \frac{1}{H}\,dV$. However, from the equilibrium we found that $dV = w dS$, which gives

$$dy' = \frac{w}{H}\,dS$$

In an infinitesimal portion dS of the line, we can use Pythagoras' theorem to obtain

$$dS = \sqrt{dx^2 + dy^2} = \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\,dx = \sqrt{1 + (y')^2}\,dx$$

Combining these two equations results in

$$dy' = \frac{w}{H}\sqrt{1 + (y')^2}\,dx$$

16 This does not means that the horizontal is *a constant*. It is constant for any particular configuration, but changes from one configuration to another.

Separating the variables we get

$$\frac{dy'}{\sqrt{1+(y')^2}}=\frac{w}{H}dx$$

In this work, we are interested in a anchor mooring for a flat horizontal water base. This means that the end at the bottom (arbitrarily set at $x=0$) has slope zero. Using this condition, we can integrate the equation above:

$$\int_0^{Y'}\frac{dy'}{\sqrt{1+(y')^2}}=\int_0^{X}\frac{w}{H}dx$$

Integrating we obtain

$$\operatorname{arsinh}(Y')=\frac{w}{H}X=\frac{X}{H/w}=\frac{X}{c}$$

Solving for Y' we get

$$\boxed{Y'=\sinh\frac{X}{c}=\frac{V}{H}}$$

This is Eq.(3) in the text body. Separating variables[17] and integrating again gives

$$\int_0^{Y}dY=\int_0^{X}\sinh\frac{X}{c}dX$$

$$\boxed{Y=c\left(\cosh\frac{X}{c}-1\right)}$$

which is Eq.(1) of the text body. From the equation $dy'=\frac{w}{H}dS$, by direct integration we get

$$\int_0^{Y'}dy'=\frac{w}{H}\int_0^{S}dS$$

$$Y'=\frac{w}{H}S=\frac{S}{c}$$

and, from the expression for Y' we obtain

$$\boxed{S=c\sinh\frac{X}{c}}$$

which is Eq.(2).

Equation (4) is immediate from the force components, and Eq.(5) is defined along this derivation.

Equation (6) results from the fact that the horizontal portion of the cable laying on the water bottom has the length L of the cable less the length S of the suspended cable, which also is the same as the distance A to the anchor less the distance X to the tangent point. That is

$$L-S=A-X$$

Separating controlled and dependent parameters gives

$$\boxed{L-A=S-X}$$

which is Eq.(6).

17 $Y'=dY/dX$

Appendix 2 – The Newton-Raphson Method

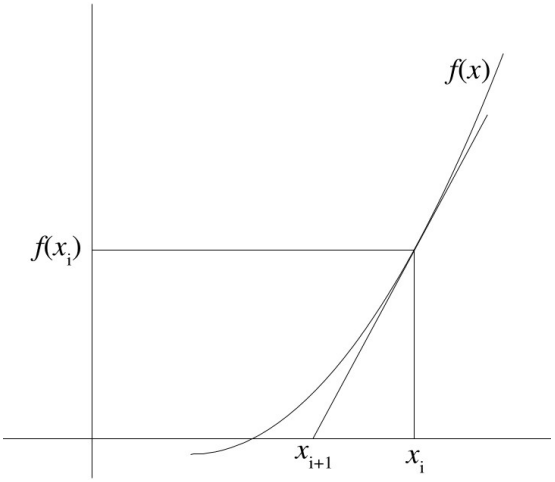
The Newton-Raphson method is a well know and simple method of root finding, which is widely used particularly for nonlinear equations. Its formulation is simple and is depicted in the figure below.

Given a function $f(x)=0$ and an initial guess x_i (or any intermediary step trial in the process), a next trial x_{i+1} is obtained by the intersection of the tangent of the curve at the point $[x_i, f(x_i)]$ and the x-axis. The tangent has slope $f'(x_i)$, and it defines the right triangle of coordinates $[x_i, 0]$, $[x_{i+1}, 0]$, and $[x_i, f(x_i)]$. By observing this triangle we note that

$$f'(x_i)=\frac{f(x_i)}{x_i-x_{i+1}}$$

Solving this expression for x_{i+1} gives:

$$\boxed{x_{i+1}=x_i-\frac{f(x_i)}{f'(x_i)}}$$



There are several possible problems with the N-R method. One, in particular is prone to occurs with the functions we are dealing with:

$$f(\xi)=\frac{\sinh\xi-\xi}{\cosh\xi-1}-k \tag{8}$$

$$f'(\xi)=1-\frac{\sinh\xi(\sinh\xi-\xi)}{(\cosh\xi-1)^2} \tag{9}$$

As said in the text body, Eq.(8) is bounded, asymptotic to $y=1-k$ and $y=-1-k$ for $+\infty$ and $-\infty$ respectively, with a single positive root for $k>0$ (which is always the case). The graph is that in Fig. (2) for $k=0.4$. For any k ($0<k<1$), Eq.(8) is symmetric with respect to the $(0,-k)$. This is a problem, because if we chose a seed sufficient larger than the root, the process may need several iteration, or enter in an unfortunate case of a continuous repetitive cycle (when a trial shoots to another trial that shoots it back to the first), or diverge to $\pm\infty$.

Consider, the example in the text, body, with $k=0.6$, the root is found at $\xi_4=2.04909698693$ (for a seed $\xi_0=1.8$. If we use any seed between 0 and 2.04909, we would end up in the same (correct) root. Even for larger seeds, like $\xi_0=4.0$ the method will find the correct root. But if we use $\xi_0=5.0$ the process will diverge to infinity and eventually crash.

To avoid that, we could choose a seed $\xi_0=0$, which certainly is less that the expected positive root, whichever it is. But selecting the seed as zero, when testing for convergency after the first iteration, a *division by zero* occurs ($\left|(1-\xi_1/0)\right|$). The suggestion is to consider the next trial, if we skip the first convergency test. Observing the values of equations (8) and (9)[18] for $\xi=0$ we have:

$$f(0)=\frac{\sinh 0-0}{\cosh 0-1}-k=-k$$

$$f'(0)=1-\frac{\sinh 0\,(\sinh 0-0)}{(\cosh 0-1)^2}=\frac{1}{3}$$

For these results, the next trial for the root using the N-R method, we get

$$\xi_1=\xi_0-\frac{f(\xi_0)}{f'(\xi_0)}$$

$$\xi_1=0-\frac{f(0)}{f'(0)}=0-\frac{-k}{1/3}=3k$$

This results indicates that, if the seed is zero or any very small positive number close to zero, the next trial will always be equal to, or slightly larger than $3k$. For example, if we a seed equal to 0.01 the next trial would be $3.00003k$. That is the reason we suggest $3k$ for the N-R seed[19].

18 For these cases, refer to the limit values obtained in the text body.

19 Besides being in the convergency range, we reduce the number of iterations by 1.